Entrance tests

Analysis

1. The trigonometric Fourier series, root-mean-square convergence, point-by-point convergence and uniform convergence.
2. The Fourier transform in $L_1$ and $L_2$. The Plancherel theorem.
4. Conjugate space. The conjugate for the Hilbert space.

The conjugates for classical functional spaces and spaces of sequences.
5. Measurable functions. Almost everywhere convergence and convergence in measure; Lebesgue and Riesz theorems. The structure of measurable functions, Luzin’s theorem.
6. The passage to the limit under the Lebesgue integral sign.
7. The space $L_p$, $1 \leq p < \infty$. Hölder’s Inequality and Minkowski’s inequality. Completeness of the space $L_p$. The density of classical sets of functions in the space $L_p[a, b]$.
10. The maximum modulus principle of an analytic function.
11. The uniqueness theorem for analytic functions.
12. The principle of the argument, Rouche’s theorem.

Differential Equations

14. The theorem of existence and uniqueness of solution of the initial problem for the systems of ordinary differential equations. Solutions in terms of Caratheodory.
15. Linear systems with periodic coefficients. The Floquet theorem. The theorem on reducibility.

Mathematical physics
26. The existence and uniqueness theorem for the classical solution of the boundary value problem for a one-dimensional wave equation on an interval.


28. The existence and uniqueness theorem for the classical solution of the boundary value problem for the one-dimensional heat equation on an interval.


30. The theorems of uniqueness and continuous dependence on the boundary functions of solutions of internal and external Dirichlet problems for the Laplace and Poisson equations.

31. The Fourier method for solving the basic boundary-value problems of mathematical physics. Steklov’s theorem.

32. The method of integral transformations (Laplace and Fourier) for solving the basic boundary value problems of mathematical physics.

33. One-dimensional heat equation. Poisson’s formula.

34. The maximum principle for equations of the parabolic type.

35. Generalised derivatives and Sobolev spaces.


38. Generalised solutions of equations of the parabolic type.

**Algebra, geometry, and mathematical logic**


41. Abelian groups. The structure of a free Abelian group. The subgroups of free Abelian groups.

42. Sylow’s theorems for finite groups.

43. Nilpotent groups. The Burnside-Wieland theorem.

44. Free groups: construction, a lemma on generating subgroups and Schreier systems, the subgroup theorem.

45. Commutative Noetherian rings. Hilbert’s basis theorem.

46. The radical of the ring. Definition and properties. The radical of the matrix ring.

47. Semi-simple Artinian rings. The first Wedderburn-Artin theorem.

48. The Maschke’s theorem.

49. The second Wedderburn-Artin theorem.

50. Lie algebras. Engel’s theorem.

51. Distributive and modular lattices. Distributivity and modularity criteria.

52. Modular lattices, the isomorphism theorem. The Kurosh-Ore theorem.


54. Geometric lattices.

**Discrete mathematics and mathematical cybernetics**

1. The inclusion-exclusion principle, the derivation of a formula for the partition number.

2. The Euler-Maclaurin formula, its derivation, the calculation of sums on its basis.

3. Stirling formula, its derivation and application.

4. Recurrence relations, theorem on the general solution for homogeneous relations with constant coefficients.

5. Euler cycle, Euler’s theorem for non-oriented and oriented cases.

6. The Hamiltonian cycle, Ore’s theorem.
7. Planar graphs, Euler’s polyhedron formula.
8. Graph planarity criterion.
9. Graph colouring, Brooks’s theorem.
10. Flat graph colouring, the Heawood theorem.
12. Complete systems of Boolean functions, closed classes, and the Post theorem.
13. Formalisation of the first-order logic (calculus of predicates), the Skolem normal form (SNF) theorem.
14. The resolution method in first-order logic.
15. The Gödel’s completeness theorem for the first-order logic.
16. Recursive functions and recursively enumerable sets.
17. Turing machines, examples of algorithmically unsolvable problems.
18. Computational complexity of the algorithm and problem; P, NP, PSPACE complexity classes.
20. Deterministic and non-deterministic finite automata, the Rabin-Scott theorem.
21. Regular languages and finite automata, the Kleene’s theorem.
22. The Myhill-Nerode theorem and the construction of a minimal deterministic automaton.
23. Context-free grammars and languages, the substitution theorem.
24. Context-free grammars and languages, a lemma on pumping.
25. MP-automata, the context-free language class recognition theorem.
26. Greedy algorithms, definition and analysis of the Kruskal’s and Dijkstra’s algorithms.
27. Divide-and-conquer algorithms and complexity estimates using the fast multiplication example.
29. Dynamic programming, definition and analysis of the complexity of the Ford-Bellman algorithm and the Knapsack algorithm.
30. Flows in graphs, the theorem and the Ford-Falkerson algorithm.
31. Information and entropy, the Shannon’s source coding theorem.
32. Statement of the linear programming problem, the simplex algorithm (or simplex method).
33. The polynomial solution of the linear programming problem.
34. The dual problem of linear programming, the duality theorem.

Theoretical mechanics

35. Canonical transformations. The Hamilton-Jacobi equation, the total integral.
36. The Gauss’s principle of least constraint. The Hertz’s principle of the least curvature. The Hamilton’s principle. The principle of least action in the forms of Lagrange and Jacobi.

44. Linear controlled systems. The maximum principle and the calculus of variations. The Bellman’s dynamic programming method. The problem of the optimal stabilisation of controlled motions. The N.N. Krasovskiy’s theorem.

Deformable solid mechanics

45. Scalar, vector, tensor of different rank in the description of physical and mechanical properties of materials. Basic operations: addition, multiplication and contraction of tensors.


Mechanics of liquid, gas and plasma

50. The principles of continuity and the model of continuous media.

51. The equation of continuity.

52. The Euler equation (inference). Equations of state.

53. The Bernoulli equation. Examples of use, problems on the flow of an ideal fluid through a pipe of variable cross-section (the Venturri tube); the flow of liquid from the vessel with the hole.

54. The law of conservation of velocity circulation (Thomson’s theorem).

55. Potential currents.

56. The Cauchy-Lagrange integral. The attached mass of the ball.

57. Newtonian and non-Newtonian fluids.

58. The Navier-Stokes equation for compressible and incompressible liquids.

59. Examples of flow of incompressible liquids, the flat currents of Couette and Poiseuille.

60. Flows at low Reynolds numbers. Stokes problem on the flow past a ball.

61. Flow past a plate with a viscous liquid (Blasius problem).


63. The Kolmogorov-Obukhov theory.

64. The turbulent boundary layer. Turbulent flow in the pipes.

Dynamics, strength of machines

68. The Hamilton-Ostrogradskii principle for elastic systems.
69. Calculation of statically indeterminate systems by the method of forces. Calculation of statically indeterminate systems by the displacement method. Calculation of statically indeterminate systems by the finite element method.
71. Unsteady modes in linear systems.
72. Methods for analysing the stress-strain state in experimental studies.