Entrance tests

Analysis

1. The trigonometric Fourier series, root-mean-square convergence, point-by-point convergence and uniform convergence.
2. The Fourier transform in L_1 and L_2. The Plancherel theorem.
3. Normed and Banach spaces. Three principles of the linear analysis (theorems of Hahn-Banach, Banach-Steinhaus, Banach's inverse operator).
4. Conjugate space. The conjugate for the Hilbert space.
The conjugates for classical functional spaces and spaces of sequences.
5. Measurable functions. Almost everywhere convergence and convergence in measure; Lebesgue and Riesz theorems. The structure of measurable functions, Luzin's theorem.
6. The passage to the limit under the Lebesgue integral sign.
7. The space L_p, $1 \leq p <$. Hölder's Inequality and Minkowski's inequality. Completeness of the space L_p. The density of classical sets of functions in the space $L_p[a, b]$.
8. Series of analytic functions. Taylor series of analytic function.
9. The Laurent series. Isolated singular points of an analytic function. Deductions, Cauchy residue theorem.
10. The maximum modulus principle of an analytic function.
11. The uniqueness theorem for analytic functions.
12. The principle of the argument, Rouche's theorem.
13. The Riemann-Schwarz symmetry principle.

Differential Equations

14. The theorem of existence and uniqueness of solution of the initial problem for the systems of ordinary differential equations. Solutions in terms of Caratheodory.
15. Linear systems with periodic coefficients. The Floquet theorem. The theorem on reducibility.
16. Autonomous systems of differential equations. The equilibrium positions. The classification of equilibrium points.
17. Limiting cycles. Bifurcation of the birth of a cycle from an equilibrium position. The Hopf theorem. Soft and hard bifurcations.
18. Lyapunov stability, asymptotic stability. The stability theorem in the first approximation.
19. Orbital stability. Stability of periodic solutions of an autonomous system. Poincaré stability criterion for an orbit.
20. Boundary value problems for linear differential equations. Self-adjoint boundary value problems. Eigenfunctions and eigenvalues of the boundary value problem. The Sturm-Liouville boundary-value problems.
21. Partial differential equations of the first order.
22. Fredholm integral equations of the second kind. The method of successive approximations. The Fredholm's theorem. Hermite kernels. The Hilbert-Schmidt theorem.
23. Improperly posed problems. Integral Fredholm equations of the first kind and methods for their regularisation. General methods for regularising improperly posed problems.
24. Calculus of variations problem with moving and fixed boundaries. Necessary conditions in the form of Euler equations. Sufficient conditions.
25. The problems of optimal control. The Pontryagin maximum principle for the problem of optimal fast action. A problem with fixed time. Relationship of the maximum principle with the dynamic programming method. The Bellman equation.

Mathematical physics

26. The existence and uniqueness theorem for the classical solution of the boundary value problem for a one-dimensional wave equation on an interval.
27. The Fourier Law. Mathematical models of heat conduction processes.
28. The existence and uniqueness theorem for the classical solution of the boundary value problem for the one-dimensional heat equation on an interval.
29. Properties of harmonic functions. The flow theorem. The mean value theorem. Extremal properties of harmonic functions.
30. The theorems of uniqueness and continuous dependence on the boundary functions of solutions of internal and external Dirichlet problems for the Laplace and Poisson equations.
31. The Fourier method for solving the basic boundary-value problems of mathematical physics. Steklov's theorem.
32. The method of integral transformations (Laplace and Fourier) for solving the basic boundary value problems of mathematical physics.
33. One-dimensional heat equation. Poisson's formula.
34. The maximum principle for equations of the parabolic type.
35. Generalised derivatives and Sobolev spaces.
36. Generalised solutions of boundary value problems for equations of the elliptic type. Theorems of existence, uniqueness and continuous dependence.
37. Generalised solutions of wave equations.
38. Generalised solutions of equations of the parabolic type.

Algebra, geometry, and mathematical logic

39. Decomposition of a group by a double module. Subgroup index. The theorems of Lagrange and Poincare. Conjugacy of complexes. Normal subgroups. Normaliser and centraliser of the complex. Number of complexes conjugate to the given one.
40. Normal and subnormal series. The Kurosh-Zassenhaus Lemma. The Schreier's theorem on isomorphic compactions of the subnormal series. The Jordan-Hölder theorem.
41. Abelian groups. The structure of a free Abelian group. The subgroups of free Abelian groups.
42. Sylow's theorems for finite groups.
43. Nilpotent groups. The Burnside-Wieland theorem.
44. Free groups: construction, a lemma on generating subgroups and Schreier systems, the subgroup theorem.
45. Commutative Noetherian rings. Hilbert's basis theorem.
46. The radical of the ring. Definition and properties. The radical of the matrix ring.
47. Semi-simple Artinian rings. The first Wedderburn-Artin theorem.
48. The Maschke's theorem.
49. The second Wedderburn-Artin theorem.
50. Lie algebras. Engel's theorem.
51. Distributive and modular lattices. Distributivity and modularity criteria.
52. Modular lattices, the isomorphism theorem. The Kurosh-Ore theorem.
53. Semimodular lattices. The Jordan-Hölder theorem.
54. Geometric lattices.

Discrete mathematics and mathematical cybernetics

1. The inclusion-exclusion principle, the derivation of a formula for the partition number.
2. The Euler-Maclaurin formula, its derivation, the calculation of sums on its basis.
3. Stirling formula, its derivation and application.
4. Recurrence relations, theorem on the general solution for homogeneous relations with constant coefficients.
5. Euler cycle, Euler's theorem for non-oriented and oriented cases.
6. The Hamiltonian cycle, Ore's theorem.

7. Planar graphs, Euler’s polyhedron formula.
8. Graph planarity criterion.
9. Graph colouring, Brooks’s theorem.
10. Flat graph colouring, the Heawood theorem.
11. Boolean functions, normal forms, and Zhegalkin polynomials.
12. Complete systems of Boolean functions, closed classes, and the Post theorem.
13. Formalisation of the first-order logic (calculus of predicates), the Skolem normal form (SNF) theorem.
14. The resolution method in first-order logic.
15. The Gödel’s completeness theorem for the first-order logic.
16. Recursive functions and recursively enumerable sets.
17. Turing machines, examples of algorithmically unsolvable problems.
18. Computational complexity of the algorithm and problem; P, NP, PSPACE complexity classes.
19. Reducibility and completeness in the P, NP, PSPACE complexity classes.
20. Deterministic and non-deterministic finite automata, the Rabin-Scott theorem.
21. Regular languages and finite automata, the Kleene’s theorem.
22. The Myhill-Nerode theorem and the construction of a minimal deterministic automaton.
23. Context-free grammars and languages, the substitution theorem.
24. Context-free grammars and languages, a lemma on pumping.
25. MP-automata, the context-free language class recognition theorem.
26. Greedy algorithms, definition and analysis of the Kruskal’s and Dijkstra’s algorithms.
27. Divide-and-conquer algorithms and complexity estimates using the fast multiplication example.
28. Fast Fourier transform.
29. Dynamic programming, definition and analysis of the complexity of the Ford-Bellman algorithm and the Knapsack algorithm
30. Flows in graphs, the theorem and the Ford-Falkerson algorithm.
31. Information and entropy, the Shannon’s source coding theorem.
32. Statement of the linear programming problem, the simplex algorithm (or simplex method).
33. The polynomial solution of the linear programming problem.
34. The dual problem of linear programming, the duality theorem.

Theoretical mechanics

35. Canonical transformations. The Hamilton-Jacobi equation, the total integral.
36. The Gauss’s principle of least constraint. The Hertz’s principle of the least curvature. The Hamilton’s principle. The principle of least action in the forms of Lagrange and Jacobi.
37. Non-holonomic systems. Possible displacements in the case of non-holonomic constraints. Equations of motion in the form of Routh, Appel. Movement of bodies over an absolutely rough surface (plane).
38. Dynamics of a rigid body. Statement of the problem of the motion of a heavy solid. First integrals. The Euler-Poinsot and Lagrange-Poisson cases. Regular precession. Permanent rotations and their stability.
39. The Poisson bracket and its properties. The Jacobi identity. Poisson's theorem on first integrals.
40. Periodic oscillations in non-autonomous nonlinear systems in the non-resonance and resonance cases. Stability of periodic oscillations in non-autonomous non-linear systems.
41. Periodic oscillations in autonomous nonlinear systems. The Lyapunov-Poincaré bifurcations and the Andronov-Hopf bifurcations. Stability of periodic oscillations in autonomous nonlinear systems. The Andronov-Witt theorem.
42. Almost periodic oscillations in non-linear systems. The Krylov-Bogolyubov averaging method. Stability of almost periodic oscillations in non-linear systems.

43. Parametric resonance. Self-oscillations. The phase plane method. The concept of the limit cycle. The Poincaré's method of the small parameter. The averaging method. A theorem on the justification of the averaging method.
44. Linear controlled systems. The maximum principle and the calculus of variations. The Bellman's dynamic programming method. The problem of the optimal stabilisation of controlled motions. The N.N. Krasovsky's theorem.

Deformable solid mechanics

45. Scalar, vector, tensor of different rank in the description of physical and mechanical properties of materials. Basic operations: addition, multiplication and contraction of tensors
46. Stress theory. Principle of stress. Stress tensor. Stress state at a point. Transformation of the stress tensor components. Invariants of the stress tensor. Expansion of the stress tensor on the ball and deviator. Stresses on sloping platforms. Conditions of equilibrium at the boundary. Cauchy's formula. Equations of equilibrium in stresses. The main stresses. Maximum tangent stresses.
47. The theory of deformations. Vector of displacements. The strain tensor. Representation of the non-linear strain tensor in terms of the linear tensor and the small-rotation tensor. The tensor of small deformation. Consistency equations. Deformed state at the point of the body. Invariants of the strain tensor and principal deformations. Maximum shift.
48. Defining relationships of the elasticity theory. The generalised Hooke's law. The Hooke's law for isotropic material. The boundary value problem. Boundary-value problems in displacements and stresses. The Lame equation. The uniqueness theorem for a solution. The generalised Hooke's law for anisotropic material. Cases of elastic symmetry of the body. Different ways of describing elastic properties and their relationship.
49. Plasticity. Theory of small elastoplastic deformations. The theorem on simple loading. The unloading theorem. The method of elastic solutions. The theory of flow. The Drucker's postulate. Associated flow law. Gradientality of the vector of increment of plastic deformations to the yield surface. Translational and isotopic hardening.

Mechanics of liquid, gas and plasma

50. The principles of continuity and the model of continuous media.
51. The equation of continuity.
52. The Euler equation (inference). Equations of state.
53. The Bernoulli equation. Examples of use, problems on the flow of an ideal fluid through a pipe of variable cross-section (the Venturri tube); the flow of liquid from the vessel with the hole.
54. The law of conservation of velocity circulation (Thomson's theorem).
55. Potential currents.
56. The Cauchy-Lagrange integral. The attached mass of the ball.
57. Newtonian and non-Newtonian fluids.
58. The Navier-Stokes equation for compressible and incompressible liquids.
59. Examples of flow of incompressible liquids, the flat currents of Couette and Poiseuille.
60. Flows at low Reynolds numbers. Stokes problem on the flow past a ball.
61. Flow past a plate with a viscous liquid (Blasius problem).
62. Turbulence. Reynolds equations.
63. The Kolmogorov-Obukhov theory.
64. The turbulent boundary layer. Turbulent flow in the pipes.

Dynamics, strength of machines

65. Free oscillations of beams. Forced oscillations of beams. Fluctuations of beams on an elastic base.

66. Equation of longitudinal vibrations of rods. Torsional vibrations of rods. Flexural vibrations of rods.
67. Numerical methods for solving problems of dynamics and strength.
68. The Hamilton-Ostrogradskii principle for elastic systems.
69. Calculation of statically indeterminate systems by the method of forces. Calculation of statically indeterminate systems by the displacement method. Calculation of statically indeterminate systems by the finite element method.
70. Longitudinal strike. Torsional impact. Impact under bending. Combined strike. Determination of dynamic coefficients upon impact.
71. Unsteady modes in linear systems.
72. Methods for analysing the stress-strain state in experimental studies.